Let $\mathcal{M}$ be a compact Riemannian manifold and let $G(x, y)$ be the Green function for the Laplacian:

$$\Delta_x G(x, y) = \delta_y(x) - V^{-1} \text{vol}$$

Define the (discrete) Green energy by

$$E_G(x_1, ..., x_N) = \sum_{i \neq j} G(x_i, x_j).$$

If $\mathcal{M} = \mathbb{S}^2$, then $G(x, y) = \log \|x - y\|^{-1}$ (essentially).
Separation distance

Theorem

For a collection of \( N \) minimal logarithmic energy points on \( S^2 \) there is a radius \( r = r(N) \) such that \( x_i \not\in B(x_j, r) \) for every \( i \neq j \).

⇒ Separation distance result.

The natural "area of influence" in a general compact manifold appears to be not a geodesic ball, but a harmonic ball.

\[
B^{\text{harm}}(p, t) = \text{the blob of } t \text{ units of fluid injected at } p.
\]